GAMMA-RAY BURST BEAMING: A UNIVERSAL CONFIGURATION WITH A STANDARD ENERGY RESERVOIR?

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ABSTRACT

We consider a gamma-ray burst (GRB) model based on an anisotropic fireball with an axisymmetric energy distribution of the form $\epsilon(\theta) \propto \theta^{-k}$, and allow for the observer's viewing direction being at an arbitrary angle θ_v with respect to the jet axis. This model can reproduce the key features expected from the conventional on-axis uniform jet models, with the novelty that the achromatic break time in the broadband afterglow lightcurves corresponds to the epoch when the relativistic beaming angle is equal to the viewing angle, θ_v, rather than to the jet half opening angle, θ_j. If all the GRB fireballs have such a similar energy distribution form with $1.5 < k \lesssim 2$, GRBs may be modeled by a quasi-universal beaming configuration, and an approximately standard energy reservoir.

Subject headings: gamma rays: bursts - shock waves - ISM: jets and outflows

1. INTRODUCTION

Recently, several independent approaches have led to the conclusion that long gamma-ray bursts (GRBs) have a standard energy reservoir of several 10^{50} ergs (Frail et al. 2001, hereafter F01; Panaitescu & Kumar 2001, hereafter PK01; Piran et al. 2001, hereafter P01). An important ingredient of this argument is that the putative jet opening angles, θ_j, as inferred from the afterglow lightcurve breaking time, t_b, have a broad distribution, but just of the right form to compensate for the wide dispersion of the "isotropic" energy emitted in γ-rays, $E_{\gamma,\rm iso}$, so that $E_\gamma = (E_{\gamma,\rm iso}/4\pi)(\theta_j^2/2)$ is essentially invariant (F01). The total energy of the fireball should be $E_{\rm tot} \geq E_\gamma + E_0$, where E_0 is the initial kinetic energy of the fireball in the afterglow phase assuming an adiabatic evolution, and the inequality takes into account the possible energy loss during the radiative regime in the early afterglow phase that has evaded the present observations. Writing $E_\gamma = \eta E_{\rm tot}$ where η is the gamma-ray emission efficiency, $E_{\rm tot}$ could be mainly contributed by E_0 if η is small (e.g. < 0.1). PK01 and P01 found that E_0 is also distributed in a narrow range. For a uniform jet, this leads to the inference that $E_0 = (dE/d\Omega)(\theta_j^2/2)$ is also essentially invariant. However, in the above analysis, and in the current afterglow jet models which are used to determine θ_j, it is generally assumed that the jets are uniform, with sharp cut-offs at the edges, and that the line-of-sight cuts right across the jet axis. Neither of these assumptions are necessarily true in general (Mészáros , Rees & Wijers 1998; McFadyen & Woosley 1999; Woods & Loeb 1999; Nakamura 1999; Salmonson 2001; Dai & Gou 2001). On the other hand, although it is not difficult to construct a central engine model which makes GRBs with a standard energy reservoir but with quite different beaming angles, it would be more elegant to have a model that all the GRB beams share a standard energy reservoir as well as *a quasi-universal beaming configuration* (M. J. Rees, 2001, private communication). Here we show that such a model is possible by taking account of the off-axis anisotropic jet effects, without violating the present observational constraints.

2. THE MODEL

Our assumption is that all the long GRBs have a quasi-universal beam configuration, with an anisotropic energy distribution along the θ direction with an axial symmetry. We define the jet axis to be physically related to the rotational axis of the central engine. In the spherical coordinate system, we assume that the angular distribution of the energy is (e.g. Mészáros et al. 1998)

$$dE/d\Omega = \epsilon(\theta, \phi) = \epsilon(\theta) = \epsilon_0 \theta^{-k}, \qquad (1)$$

within the range $\theta_0 \leq \theta \leq \Theta$, where θ_0 is a very small angle within which some deviation from (1) is necessary to avoid the divergence at $\theta = 0$, and Θ is some large angle which exceeds the presently measured θ_j by at least a factor of 2 (for the simplification of the discussions below). In principle, the jet configuration is time-dependent, due to effects such as the eventual sideways expansion (Rhoads 1997, 1999), but we assume that such changes are not prominent when $\Gamma(\theta_v) \geq 1/\theta_v$, where θ_v is the observer's viewing angle with respect to the jet axis (which is θ_j in the conventional model). By adopting this assumption, we have also implicitly assumed that the angular-dependence of the baryon loading rate is also weak so that it does not modify the $\epsilon(\theta)$ profile, so that the Lorentz factor angular distribution follows a similar law, i.e., $\Gamma(\theta) \propto \theta^{-k}$. Furthermore, we demand

$$(1.5) < k \lesssim 2. \qquad (2)$$

The reason for this requirement will become evident later. The main conjecture of the model is that *the dispersion in the breaking time, t_b, in the afterglow data is a manifestation of the diverse viewing angles of the observers, rather than the diverse intrinsic opening angles of the jets themselves.* In other words, what were inferred by Frail et al. (2001) as θ_j are θ_v in our model. We will test whether the above hypothesis is able to pass the following three criteria: (1) When $\Gamma(\theta_v) \gg 1/\theta_v$, the jet dynamics along the line-of-sight satisfies the isotropic law $\bar{\Gamma}(\theta_v, t) \propto t^{-3/8}$ (for simplicity, we only discuss an adiabatic fireball running into an interstellar medium with a constant density), where t is the observer time, and $\bar{\Gamma}(\theta_v, t)$ is an effective Lorentz factor assuming an isotropic fireball which could mimic the emission in the direction θ_v at the time t; (2) When $\Gamma(\theta_v) < 1/\theta_v$, the dynamics changes so that the lightcurves are steepened, and in the asymptotic limit, the dynamics is $\bar{\Gamma}(\theta_v, t) \propto t^{-1/2}$, as a result of the sideways expansion (Rhoads 1999, Sari, Piran & Halpern 1999); (3) The total jet energy $E_{\rm tot}$ is essentially a universal value.

For an isotropic adiabatic fireball running into a uniform medium, $\Gamma(t) \propto \epsilon^{1/8} n^{-1/8} (at)^{-3/8}$, where $\epsilon = dE/d\Omega$ is the energy per solid angle, n is the ambient medium number density, and the blastwave radius is written in a general form as $R = a\Gamma^2 ct$, where the factor a effectively takes into account the surface of equal-arriving-time as well as the thickness of the emitting region. To test the criteria (1) and (2), which are the basic features of the present jet models, the key is to estimate the effective energy per solid angle, $\bar{\epsilon}(\theta_v, \phi_v, t) = \bar{\epsilon}(\theta_v, t)$, in the direction (θ_v, ϕ_v), and to evaluate the possible time-dependence of this value. When $\Gamma(\theta_v, t) = \Gamma > 1/\theta_v$, the observer can only observe a solid angle around (θ_v, ϕ_v) with a half opening angle of order $1/\Gamma$. By definition, the effective energy per solid angle in the direction (θ_v, ϕ_v) is

$$\begin{aligned} \bar{\epsilon}(\theta_v, \phi_v, t) &= \frac{\int_{\theta_v-1/\Gamma}^{\theta_v+1/\Gamma} \epsilon(\theta, t)\sin\theta d\theta \int_{\phi_v-1/\Gamma}^{\phi_v+1/\Gamma} d\phi}{\int_{\theta_v-1/\Gamma}^{\theta_v+1/\Gamma} \sin\theta d\theta \int_{\phi_v-1/\Gamma}^{\phi_v+1/\Gamma} d\phi} \\ = \bar{\epsilon}(\theta_v, t) &= \frac{\int_{\theta_v-1/\Gamma}^{\theta_v+1/\Gamma} \epsilon(\theta, t)\sin\theta d\theta}{\int_{\theta_v-1/\Gamma}^{\theta_v+1/\Gamma} \sin\theta d\theta} \end{aligned} \quad (3)$$

In the small angle approximation, which is relevant to the present discussions[1], one has $\sin\theta \sim \theta$. With (1), this gives

$$\bar{\epsilon}(\theta_v, t) = \frac{\int_{\theta_v-1/\Gamma}^{\theta_v+1/\Gamma} \epsilon_0 \theta^{1-k} d\theta}{\int_{\theta_v-1/\Gamma}^{\theta_v+1/\Gamma} d\theta} \simeq \epsilon_0 \theta_v^{-k} = \epsilon(\theta_v), \quad (4)$$

where the condition for the approximate equality is $\Gamma \gg 1/\theta_v$. This is a time-independent quantity, since the sideways expansion effect is not important at the same stage. We then have

$$\bar{\Gamma}(\theta_v, t) \propto [\bar{\epsilon}(\theta_v, t)]^{1/8} n^{-1/8} (at)^{-3/8} \propto t^{-3/8}, \quad \Gamma \gg 1/\theta_v. \quad (5)$$

This indicates that the observer does not feel the anisotropy of the fireball when the relativistic beaming angle $1/\Gamma$ is much smaller than the viewing angle θ_v, but observes the fireball as if it were isotropic. This is the same conclusion as drawn in the on-axis uniform jet model. Notice that the above conclusion does not require (2). In fact, it holds for any k value. One thing to notice is that the factor a may deviate from the conventional value (e.g. ~ 4), since the shape of the equal-arrival-time surface will be distorted due to the anisotropic distribution of the fireball energy. However, its time-dependence, if any, would be very small. Therefore, although it may influence the absolute values of the afterglow flux levels, such an effect does not change the blastwave dynamics in the viewing direction.

When the blastwave decelerates so that the line-of-sight bulk Lorentz factor $\Gamma(\theta_v)$ drops to and below $1/\theta_v$, the dynamics along the line-of-sight will change, i.e., $\bar{\Gamma}(\theta_v)$ will deviate from the $\propto t^{-3/8}$ dependence. Just like in the on-axis uniform jet model, two effects play a role here. First, as $1/\Gamma(\theta_v)$ starts to exceed θ_v, the observer starts to feel the energy deficit due to the sudden drop of the energy distribution on the other side of the jet axis. Although the calculation of $\bar{\epsilon}(\theta_v, t)$ is no longer straightforward, this deficit effect should mimic that in the uniform jet model as long as k is not too flat, say, $k > 1.5$. Second, the anisotropic jet has a trend to retain the isotropic shape as a result of expanding sideways. The expansion center is at the jet axis, where the energy is mostly concentrated. The sideways expansion angular scale increases with time as $\theta_{ex} = c_{ex} t'/R \sim (c_{ex}/c)/\Gamma(\theta_{ex})$, where $t' \sim R/c\Gamma(\theta_{ex})$ is the co-moving time, which is essentially defined by the Lorentz factor at the expansion edge, $\Gamma(\theta_{ex})$, since this is the smallest value (hence, it defines the longest co-moving time). The expansion speed c_{ex} may be either the relativistic speed of sound $\sim c/\sqrt{3}$ (Rhoads 1999), or simply the speed of light (Sari et al. 1999). In any case, at the viewing direction θ_v, the observer will start to feel a stronger deceleration of the blastwave due to the sideways expansion within the cone defined by θ_v, as $\Gamma(\theta_v)$ drops below $1/\theta_v$, although the global sideways expansion will become evident only when $\Gamma(\theta_v)$ drops below $1/\Theta$. It is certainly necessary to model in detail the time it takes to achieve an exponential slowing down, but in the asymptotic phase when this occurs, R is essentially a constant, and one should have $\bar{\Gamma}(\theta_v, t) \propto t^{-1/2}$ eventually (from $t \sim R/\bar{\Gamma}^2$). In this regime, the temporal indices of the lightcurves in the various spectral regimes follow closely the same predictions as in the uniform jet models (Sari et al. 1999; Rhoads 1999). For example, in the slow-cooling regime (which is usually the case after the viewing or "jet" break), for spectral regimes both below and above the cooling frequency, one should have the spectral flux $F_\nu \propto t^{-p}$, where p is the power-law index of the electron number distribution. For reasonable values of p (e.g. ~ 2.2), this is consistent with several GRB afterglow observations. One expectation of this model is that, the smaller θ_v, the longer it takes to reach the asymptotic exponential deceleration regime. This seems to be the case of GRB 990123, which has a very small θ_v (PK01, F01), and the lightcurve slope beyond the break seems to be not steep enough to meet the aymptotic jet temporal index.

We have shown that the present model can reproduce the key features of the on-axis uniform jet model, with an arbitrary k value as long as it is not too flat. The remaining question is whether the model can also retain the merit of a standard energy reservoir invoked in the conventional jet model. In principle, one does not have to fulfill this constraint, but just wishes so for the sake of elegance. By definition, the total energy in the fireball is

$$E_{\rm tot} = 2\pi \int_0^\Theta \epsilon(\theta)\sin\theta d\theta \simeq 2\pi \int_{\theta_0}^\Theta \epsilon_0 \theta^{1-k} d\theta. \quad (6)$$

For $k < 2$, we get

$$E_{\rm tot} = \frac{2\pi}{2-k} b^{2-k} \epsilon(\theta_v) \theta_v^2, \quad (7)$$

[1]The largest "jet" angle in Frail et al. (2001) is 0.411, and the approximation is good within 3%.

where we have parameterized $\Theta = b\theta_v$. We can see that the quantity $2\pi\epsilon(\theta_v)\theta_v^2$ (which is essentially E_γ of F01, or E_0 of PK01 and P01) is quasi-invariant, if k and $E_{\rm tot}$ are constant (or with a small scatter). The only extra scatter is introduced through the scatter of b, which is introduced by the scatter of θ_v (assuming the same Θ for all GRBs). However, for the index $(2-k)$ this scatter is greatly reduced if k is not much smaller than 2. This is another reason why we require, say, $k > 1.5$, in (2). A smaller Θ can also reduce the b scatter. Notice that the b scatter tends to raise $E_{\rm tot}$ in GRBs with small θ_v's (and hence larger b's), which seems to be helpful to reduce the E_0 scatter in PK01. On the other hand, k cannot approach the limit of 2 if one is to avoid a divergence in equation (7) that could result in an unreasonably large $E_{\rm tot}$. This problem of course does not arise if Θ is small, motivated on physical grounds. An important implication of (7) satisfying such a constraint is that the total energy reservoir is standard, but the absolute value need no longer necessarily be several times 10^{50} ergs, but would depend on the value of k and the typical value of b. Given reasonable values, $E_{\rm tot}$ could be one order of magnitude higher than that of F01 and PK01, but this could be still well accommodated within conventional central engine models (Mészáros , Rees & Wijers 1999).

For $k \geq 2$, generally $E_{\rm tot}$ (eq.[6]) can not be expressed in terms of $\epsilon(\theta_v)\theta_v^2$, since most energy is distributed at small angles. The standard energy budget argument no longer holds[2]. A quasi-universal beaming configuration as well as a standard energy reservoir is however in general obtained if the requirement (2) is satisfied.

3. SUMMARY & DISCUSSION

We have shown that an off-axis anisotropic jet with the energy distribution given by equation (1) can reproduce the key observational features of the conventional on-axis uniform jet model. The novelty here is that the achromatic break time t_b in the broadband afterglow lightcurves no longer corresponds to the time when the relativistic beaming angle is equal to the jet half opening angle, θ_j. Rather, it corresponds to the time when the relativistic beaming angle is equal to the observer's viewing angle, θ_v. In this model, the broad distribution of t_b in the data is no longer due to the intrinsic scatter of the jet opening angles among different bursts, but is attributed to the distribution of the observer's lines of sight. If (2) is satisfied, all the GRBs may have a quasi-universal beaming configuration, besides a quasi-standard energy reservoir. We deem this to be a more elegant picture than the conventional on-axis uniform jet model. In addition, the conventional model is more idealized, and the present model may give a closer representation of what could be expected in nature.

Our model predictions on the afterglow lightcurves are not, however, completely equivalent to those of the uniform jet model. The key difference should occur in the "jet break" regime. Our model should give a more gradual variation at the break than the uniform jet model, which assumes a sharp drop off at the jet edge. The so far not well-studied sideways expansion effect in an anisotropic jet may further complicate the problem. The shape of the break should also depend on some unknown parameters, such as k. Detailed modeling is necessary in order to constrain this. In any case, the gradual break predicted in our model is not inconsistent with several well studied afterglow lightcurves, and some detailed simulations have shown that the conventional jet models usually also give gradual and smooth jet breaks (e.g. Panaitescu & Mészáros , 1999; Moderski, Sikora & Bulik 2000; Huang et al. 2000). Both models are compatible with the present data, but this situation may change if better data becomes available.

Recently, Rossi, Lazzati & Rees (2001) have independently discussed a similar model in more detail. They plotted the afterglow lightcurves for the $k = 2$ case which mimic those of the on-axis uniform jet model, and also discussed the more general cases of $k \neq 2$. Here we have presented a general analytical argument, showing that the blastwave dynamics at the line of sight is identical to the uniform jet model in the asymptotic regime for a locus of models of the general form of equation (1), as long as k is not too flat. With this particular form of the angular dependence of the energy, in order to have a standard, finite energy reservoir for all bursts one requires the constraint (2). The upper end $k \lesssim 2$ of the constraint (2) ensures that the total energy could be expressed in terms of $\epsilon(\theta_v)\theta_v^2$ and does not diverge. (However, the case $k \gtrsim 2$ could also have the same merit if θ_0 is not too small compared with θ_v.) The lower end of the constraint, $k > 1.5$, ensures that the scatter introduced by b is not too large, and that the energy-deficit effect at the other side of the jet is not too small.

For ease of discussion, we have here assumed $\Theta > 2\theta_v$ in §2. This is to avoid that the observer feels the energy deficit beyond Θ before the relativistic beaming angle exceeds θ_v. Indeed, if $\Theta < 2\theta_v$, $\bar{\Gamma}(\theta_v)$ starts to deviate from the value predicted by the adiabatic law $\propto t^{-3/8}$ after it is less than $(\Theta - \theta_v)^{-1}$. In this regime, the upper limits for θ-integration in the numerators of both (3) and (4) should be replaced by Θ. Thus the maximum correction factor with respect to the $\Theta > 2\theta_v$ case is a factor of $\Theta/2\theta_v$. Even for $\Theta = \theta_v$ (i.e., the line of sight marginally cuts the jet edge), the deviation is at most a factor of 1/2. We therefore conclude that the Θ effect may not be important in most cases. The main reason is that the large angles contribute a small portion to the total energy in the beam due to the distribution of the form (1).

One natural consequence of this model is that the break time t_b distribution, and hence the θ_v distribution, should satisfy the statistical distribution function due to the viewing angle effect. The present data are clearly not enough to perform such a test. More importantly, there is an observational bias against detecting bursts and afterglows with larger θ_v, due to a smaller $\epsilon(\theta_v)$ involved. Such a bias also depends on the value of k. Nevertheless, better constraints may be set as adequate data are accumulated, with the help of more detailed modeling.

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[2]If θ_0 is not too small (e.g. a not very small fraction of θ_v), however, the case $k \gtrsim 2$ could still retain the feature of a standard, finite energy reservoir.

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